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"Comparisons of Transit Stop Spacing Policies"

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An Analysis of Transit Stop Spacing Policies

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Abstract

Given a number of stops and line length, where should the stops be set for a transit route? With mathematical derivations, this paper compare three transit performance indicators under different transit stop spacing policies: ridership, transit round trip travel time and passenger walking distance. A empirical study is also presented to support the results of the theoretical derivations. Four policy implications for transit stop spacing are suggested in the paper: (1) transit operators would prefer equal demand spacing rather than equal spacing policy if the demand function is convex; (2) transit round trips travel time and operator cost under equal spacing policy would be less than that of equal demand spacing policy; (3) total passenger walking distance under equal spacing and equal demand spacing would be the same; and (4) if the travel demand is equally distributed total walking distance under equal spacing (or equal demand spacing) can reach minimum total walking distance.

1. INTRODUCTION

The purpose of this paper is to clarify some of the arguments about transit stop spacing policies. Given a number of stops and line length, four transit spacing policies related to travel demand have been proposed in the literature: (1) uniform equal spacing: stops are equally spaced along the transit line irrespective of travel demand (Holroyd, 1965; Lesley, 1976); (2) inverse demand spacing: spacings of transit stops should be inversely related to the ratio of passenger origins and destinations to volume of passengers traveling through an area (Vuchic, 1981); (3) inverse square root demand spacing: spacings of transit stops in a linear transit route should be inversely proportional to the square root of the number of passenger boarding and alighting[1]. (Vaughan and Cousins, 1977; Webster and Bly, 1979; Kush and Perl, 1988; Wirasinghe and Ghoneim, 1981); and (4) equal demand spacing: the number of boarding and alighting passengers is equal for each stop.

With mathematical derivation, we investigate some of the properties of spacing policies and discuss general characteristics. Given the number of stops (determined by the budget), three interesting questions related to transit spacing policies are the followings:

1. Which spacing policy will attract more transit passengers and operator revenue?

[1] 'square root' on a linear route should be replaced by 'cube root' in a two-dimensional city (Vaughan, 1986).

2. Which spacing policy will achieve the minimum transit round trip travel time and operating cost?

3. From viewpoint of passengers, which spacing policy will minimise total access time to transit stops?

Sections 2 to 4 discuss these three issues respectively. A empirical study is presented in section 5. Finally, some policy implications are made in the last section.

2. TRANSIT TRAVEL DEMAND (OPERATOR REVENUE)

Transit travel demand (modal split) is a function of travel times and travel costs on all transport modes. There are two reasons why we could assume that all variables are constant, with the exception of walking time, in the travel demand function for comparing travel demand under different spacing policies.

1. Auto travel time, auto travel cost and transit fare are independent of spacing policy.

2. In designing bus stop locations, access time (walking time to bus stop) is the only significant factor influencing transit travel time among the four components of passenger travel time.

Thus the travel demand model could be simplified as a function of walking distance for comparing travel demand under different spacing policies.

$$D_i = f(w) \tag{1}$$

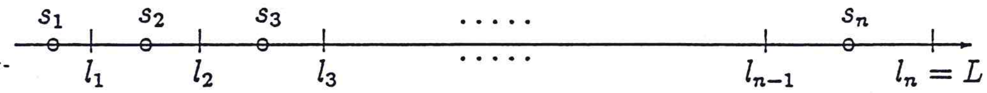

Figure 1: Transit Stops Along the Route

where f is travel demand function, and D_i is travel demand (including auto and transit) at stop i and w is walking distance.

Consider a given transit line with line length L as shown in Figure 1, for which $\{s_i, i = 1, 2, ..., n\}$ are distances from the first stop to stops $\{S_i, i = 1, 2, ..., n\}$ and $\{l_i, i = 1, 2, ..., n\}$ are the hinterland boundaries of the stops, e.g. trips started or finished between l_{i-1} and l_i will use stop S_i. Under equal spacing policy, total transit passenger demand is

$$\sum_{i=1}^{n} D_i f(\frac{L}{n}) = D_T f(\frac{L}{n}) \tag{2}$$

where D_T is total travel demand. L is transit line length. On the other hand, total transit travel demand under equal demand policy is

$$\sum_{i=1}^{n} \frac{D_T}{n} f(l_i - l_{i-1}) = \frac{D_T}{n} \sum_{i=1}^{n} f(l_i - l_{i-1}) \tag{3}$$

According to Jensen's Inequality[2] (see Mitrinovic, 1964) we obtain

$$\begin{aligned} f(\tfrac{L}{n}) &< \tfrac{1}{n}\textstyle\sum_{i=1}^{n} f(l_i - l_{i-1}) && \text{if } f(w) \text{ is strictly convex.} \\ &= \tfrac{1}{n}\textstyle\sum_{i=1}^{n} f(l_i - l_{i-1}) && \text{if } f(w) \text{ is linear.} \\ &> \tfrac{1}{n}\textstyle\sum_{i=1}^{n} f(l_i - l_{i-1}) && \text{if } f(w) \text{ is strictly concave.} \end{aligned} \tag{4}$$

[2]For every convex function $f(x)$,

$$f(\frac{1}{n}\sum_{i=1}^{n} a_i) \leq \frac{1}{n}\sum_{i=1}^{n} f(a_i)$$

In other words, equal demand spacing would attract more passengers if the $f(w)$ were strictly convex and less if it were concave. Transit operators would prefer equal demand spacing if the $f(w)$ were convex because it would attract more passengers and hence increase revenue. On the other hand, equal spacing would be preferred by transit operators if $f(w)$ were concave.

3. TRANSIT ROUND TRIP TRAVEL TIME (OPERATING COST)

Three components of a transit travel time (from the first stop to the last) are included: passenger boarding and alighting time, constant-speed travel time, and the additional acceleration and deceleration time (lost time for stopping). Among them, the transit additional acceleration and deceleration time is the only factor affected by the different transit stop spacing policies.

Suppose the spacings are long enough that transit vehicles can accelerate to maximum speed. The expected value of lost time from the first stop to the last is

$$\sum_{i=1}^{n} \frac{V_m}{2}\left(\frac{1}{r_a} + \frac{1}{r_b}\right)[1 - \exp(-D_{bi})] \qquad (5)$$

(see Ling and Taylor, 1988), where D_{bi} is the transit demand at i, V_m is the maximum transit speed, r_a and r_b are transit acceleration and deceleration rate. For equal demand spacing policy, total expected lost time from the first stop to the last is

$$\frac{V_m}{2}\left(\frac{1}{r_a} + \frac{1}{r_b}\right)\sum_{i=1}^{n}[1 - \exp(\frac{-D_{bi}}{n})] = \frac{V_m}{2}\left(\frac{1}{r_a} + \frac{1}{r_b}\right)[n - n\exp(\frac{1}{n}\sum_{i=1}^{n} -D_{bi})] \qquad (6)$$

On the other hand, for equal spacing policy,

$$\frac{V_m}{2}(\frac{1}{r_a}+\frac{1}{r_b})\sum_{i=1}^{n}[1-\exp(-D_{bi})]=\frac{V_m}{2}(\frac{1}{r_a}+\frac{1}{r_b})[n-\sum_{i=1}^{n}\exp(-D_{bi})] \qquad (7)$$

Since the negative exponential function is convex, we could obtain (8) according to Jensen's Inequality.

$$n\exp(\frac{1}{n}\sum_{i=1}^{n}-D_{bi})\leq\sum_{i=1}^{n}\exp(-D_{bi}) \qquad (8)$$

This expression indicates that expected lost time under equal demand spacing policy would greater than that under equal spacing policy. The shorter transit round trip travel time allow for small fleet size. In other words, transit round trips travel time as well as operating cost under equal demand spacing policy would greater than that under equal spacing policy.

4. PASSENGER WALKING DISTANCE

Let us further specialise to the case where all transit passengers access transit stops by walking, the average walking distance to the nearest transit stop is approximately one-fourth of the distance between two stops. Therefore the total walking distance is given by:

$$W=\frac{1}{4}\sum_{i=1}^{n}(l_i-l_{i-1})[g(l_i)-g(l_{i-1})] \qquad (9)$$

where W is the total walking distance and g is the passenger demand as a continuous function of the distance from the first stop.

Under equal spacing policy,

$$W=\frac{1}{4}\sum_{i=1}^{n}\frac{L}{n}[g(l_i)-g(l_{i-1})]=\frac{LD_b}{4n} \qquad (10)$$

where D_b is the total transit travel demand.

On the other hand, walking distance under equal demand policy is

$$W = \frac{1}{4}\sum_{i=1}^{n}(l_i - l_{i-1})\frac{D_b}{n} = \frac{LD_b}{4n} \tag{11}$$

The results of equations (10) and (11) show that total walking distance under equal spacing and equal demand spacing would be equal.

Suppose the travel demand is equally distributed, i.e. $g(x) = k$, where k is a constant. Then,

$$W = \frac{k}{4}\sum_{i=1}^{n}(l_i - l_{i-1})^2 \tag{12}$$

Under equal spacing,

$$W = \frac{k}{4}n[\frac{1}{n}\sum_{i=1}^{n}(l_i - l_{i-1})]^2 \tag{13}$$

According to Jensen's Inequality we obtain

$$n[\frac{1}{n}\sum_{i=1}^{n}(l_i - l_{i-1})]^2 \leq \sum_{i=1}^{n}(l_i - l_{i-1})^2 \tag{14}$$

In other words, if the travel demand is equally distributed then equal spacing (as well as equal demand spacing) can achieve minimum total walking distance. In this case, passengers would prefer equal spacing policy (or equal demand spacing policy). Vaughan and Cousins (1977) made the same conclusion by using a continuous model which described the trip origins and destinations along the bus route as a continuous function of distance from the first stop. The model was solved numerically to obtain the optimum bus stop spacing.

For the general case of minimising total walking distance, we take derivatives of equation (9) with respect to $l_i, i = 1, 2, ..., n-1$, set them equal to zero, and solve simultaneously,

$$\begin{cases} \frac{\partial W}{\partial l_1} = \frac{1}{4}[2g(l_1) - g(l_0) - g(l_2) + (2l_1 - l_0 - l_2)g'(l_1)] = 0 \\ \frac{\partial W}{\partial l_2} = \frac{1}{4}[2g(l_2) - g(l_1) - g(l_3) + (2l_2 - l_1 - l_3)g'(l_2)] = 0 \\ \vdots \\ \frac{\partial W}{\partial l_{n-1}} = \frac{1}{4}[2g(l_{n-1}) - g(l_{n-2}) - g(l_n) + (2l_{n-1} - l_{n-2} - l_n)g'(l_{n-1})] = 0 \end{cases} \tag{15}$$

This yields a set of $n-1$ nonlinear equations. Since l_n and l_0 are given, the equations contain $n-1$ unknown variables. Thus we can find a feasible solution $\{l_i, i = 1, 2, ..., n-1\}$ such that the total walking distance is minimum. There are several computer packages (e.g. IMSL) for solving such a system of nonlinear equations. However, the set of equations above has a special structure. Each equation contains only three unknown variables with the exception of the first and the last equations which contain only two variables each. The generalised algorithm of solving nonlinear equations is inefficient, or perhaps unable to find the global optimum solution for a large number of equations. In this paper we develop a special method to solve those equations. It is derived in following steps:

1. assume an initial value of l_1 to solve l_2 through the first equation;
2. since l_1 and l_2 are determined, we could obtain l_3 by solving the second equation. Similarly, l_4, l_5, ..., l_n could be obtained.
3. l_n should be approximately equal to L. If the differences between l_n and L are too large, set new value of l_1 and repeat step 1 and 2 until satisfactory convergence is reached.

Compared with the IMSL subroutine, this method has proved more efficient (less computer CPU time) in various numerical examples, while the results are almost identical.

5. EMPERICAL STUDY

We compare total walking distance and transit travel time under six spacing policies from the empirical results. Those spacing policies are existing system, minimum walking distance spacing, equal spacing, equal demand spacing, inverse demand spacing, and inverse square root demand spacing.

The bus route chosen for this study is the Melbourne Bus Route 700 which begins at Mordialloc, runs through Warrigal Road, and ends at Box Hill. It is 25.2 km long containing 97 bus stops. The average bus stop spacing is approximately 260 metres. A bus trip O-D survey was carried out in 1985 by Denis Johnson & Associates Pty Ltd under contract to the MTA. A total of 1811 single journeys were recorded for the route. 30.3 per cent of the trips occurred during the peak hour (4pm to 5pm). The calculations in the following are based on the peak hour. Denis Johnson & Associates (1987) discussed the details of the data collection method and procedures.

The continuous cumulative function of boarding and alighting passengers, $g(x)$, is assumed to be represented by polynomial function of the distance from the first stop. It can be calibrated from the observed bus trip O-D survey mentioned above.

The hinterland boundaries of the transit stop locations, $\{l_i, i = 1, 2, ..., n\}$, under different spacing policies are calculated as follow.

Equal Spacing

$$l_i = \frac{iL}{n} \qquad i = 1, 2, ..., n \tag{16}$$

Equal Demand Spacing

$$g(l_i) - g(l_{i-1}) = \frac{g(L)}{n} \qquad i = 1, 2, ..., n \tag{17}$$

Since l_0 is given, we can obtain $\{l_i, i = 1, 2, ..., n\}$ by solving the set of equations.

Inverse Demand Spacing

$$\begin{cases} (l_1 - l_0)[g(l_1) - g(l_0)] = (l_2 - l_1)[g(l_2) - g(l_1)] \\ (l_2 - l_1)[g(l_2) - g(l_1)] = (l_3 - l_2)[g(l_3) - g(l_2)] \\ \vdots \\ (l_{n-1} - l_{n-2})[g(l_{n-1}) - g(l_{n-2})] = (l_n - l_{n-1})[g(l_n) - g(l_{n-1})] \end{cases} \tag{18}$$

The method of solving minimum walking distance as mentioned as in section 4 can be applied to find the solutions for the set of equations above.

Inverse Square Root Demand Spacing

$$\begin{cases} (l_1 - l_0)\sqrt{g(l_1) - g(l_0)} = (l_2 - l_1)\sqrt{g(l_2) - g(l_1)} \\ (l_2 - l_1)\sqrt{g(l_2) - g(l_1)} = (l_3 - l_2)\sqrt{g(l_3) - g(l_2)} \\ \vdots \\ (l_{n-1} - l_{n-2})\sqrt{g(l_{n-1}) - g(l_{n-2})} = (l_n - l_{n-1})\sqrt{g(l_n) - g(l_{n-1})} \end{cases} \tag{19}$$

Similar method of minimum walking distance as mentioned as in section 4 can be applied to find the solutions, $\{l_i, i = 1, 2, ..., n\}$.

If people walk to the nearest bus stop, (Lesley, 1976; Wirasinghe and Ghoneim, 1981; Kikuchi, 1985) then bus stops should be located at the midpoint between boundaries, i.e. $s_i = \frac{1}{2}(l_{i-1} + l_i)$.[3] Since the transit stop locations are determined we could calculate transit demand for each stop.

$$D_{bi} = D(l_i) - D(l_{i-1}) \tag{20}$$

The results of bus stop locations and travel demand are shown in Table 1. As a simplified demonstration, only the first 15 of 97 stop locations are listed. By examining Table 1, it is found that the variations of distance between two adjoining stops under inverse demand and inverse square root demand policies lie between those of equal spacing and equal demand spacing policies.

It should be noted that the transit stops are not on the actual locations specified in Table 1. There are many factors affecting the actual location of transit stops, such as intersection location, main sites of trip generation and attraction, built form of the area, geometric design of the route and traffic signal coordination. For more details, see Institute of Traffic Engineers (1967) and Terry and Thomas (1971). For practical application, the results in Table 1 should be adjusted by these factors.

Since the transit stop locations are determined we could calculate transit demand for each stop. The total walking distance and bus travel time could be also obtained (see Ling, 1987). The results are shown in Table 2. They suggest four findings.

[3]Some papers (Vuchic and Newell, 1968; Black, 1978; Hurdle and Wirasinghe, 1980) assume people minimise travel time rather than minimise walking distance, then S_i is greater than but nearly equal to $\frac{1}{2}(l_{i-1} + l_i)$, i.e. people will have the same propensity to walk to the stop nearer their destination.

Table 1: The First 15 Bus Stop Locations and Travel Demand Under Different Spacing Policies

	Existing System		Equal Spacing		Equal Demand Spacing		Inverse Demand Spacing		Inverse Square Root Demand Spacing	
i	l_i^{**}	D_{bi}^*	l_i^{**}	D_{bi}^*	l_i^{**}	D_{bi}^*	l_i^{**}	D_{bi}^*	l_i^{**}	D_{bi}^*
1	0.30	8	0.26	7	0.56	16	0.38	10	0.34	9
2	0.60	9	0.52	8	1.02	16	0.74	11	0.67	10
3	0.90	10	0.78	8	1.43	16	1.08	12	0.99	11
4	1.20	11	1.04	9	1.79	16	1.40	12	1.29	12
5	1.65	18	1.30	10	2.12	16	1.71	13	1.59	12
6	2.10	21	1.56	11	2.44	16	2.00	13	1.88	13
7	2.30	10	1.82	11	2.73	16	2.29	14	2.16	13
8	2.50	10	2.08	12	3.02	16	2.57	14	2.44	14
9	2.85	20	2.34	13	3.29	16	2.84	15	2.71	14
10	3.20	12	2.60	13	3.55	16	3.11	15	2.98	15
11	3.40	18	2.86	14	3.81	16	3.37	15	3.24	15
12	3.60	19	3.12	15	4.06	16	3.63	15	3.51	15
13	3.90	26	3.38	15	4.30	16	3.88	16	3.77	15
14	4.20	26	3.64	16	4.55	16	4.13	16	4.02	16
15	4.60	27	3.90	16	4.79	16	4.38	16	4.28	16

* Boarding and alighting passengers.

** The hinterland boundaries of stop locations (unit: km).

Table 2: Comparison of Performances Under Different Spacing Policies

	Total Walking Distance (Km)	Transit Travel Time (min.)
Existing System	119.5	61.0
Minimum Walking Distance Spacing	95.7	61.9
Equal Spacing	98.0	61.6
Equal Demand Spacing	98.0	61.9
Inverse Demand Spacing	95.7	61.9
Inverse Square Root Demand Spacing	96.0	61.8

1. The results support two theoretical derivations in sections 3 and 4: (1) same total walking distances under equal spacing and equal demand spacing; and (2) transit travel time under equal spacing policy would be less than that of equal demand spacing policy.

2. The relocation of stop locations by the minimum walking distance spacing method could reduce total walking distance in the existing system by up to 22 per cent.

3. Although the existing system has the minimum transit total travel time, the difference between this travel time and those of the theoretical spacing policies is not significant (0.9 mim in 61 min, i.e. 1.5 per cent). This should be compared to the significant improvements in access time (about 22 per cent decrease in walking distance). Given Webster and Bly's (1979) study indicating that travellers value walking and waiting times as about twice as important as riding time, the model results suggest that there is considerable opportunity for service improvement.

4. The results of stop locations and transit performances under minimum walking distance spacing are almost the same as those of inverse demand spacing policy. These policies offer the best levels of access for passengers in this example.

6. CONCLUSIONS

Although a rigorous assessment of transit stop spacing policy needs requires detailed information about population distribution and travel demand functions, the following spacing policy implications can be drawn from the preceeding sections:

1. Transit ridership and operator revenue under different spacing policies are dependent on the travel demand function, which is related to walking distance. Transit operators would prefer equal demand spacing rather than equal spacing policy if the function is convex, otherwise equal spacing would be applied by the transit operator. Given the passenger demand function and bus stop spacing (or number of stops), a transit operator could determine bus stop spacing for more revenue.

2. Transit round trips travel time and operator cost under equal spacing policy would be less than that of equal demand spacing policy. Transit operators would prefer this policy as it offers lower operating costs. However, it does not appear significant.

3. Suppose all transit passengers access transit stops by walking only, then total passenger walking distance under equal spacing and equal demand spacing would be equal. This assumption is valid for many suburban bus routes.

4. If the travel demand is equally distributed, total walking distance under equal spacing (or equal demand spacing) can reach minimum total walking distance. Otherwise, the method presented in section 4 is proposed to find transit stop locations for minimising total walking distance. Thus passengers may prefer equal bus spacing policy within the CBD.

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